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Abstract—In this letter, we propose a novel power splitting (PS) protocol for energy harvesting (EH) cooperative communication systems. In our proposed system, the relay harvests energy from the source transmissions, by employing adaptive PS protocol, for powering the retransmissions to the destination. Unlike existing PS protocols, which are not optimal in terms of capacity maximization, our proposed adaptive PS protocol maximizes the achievable capacity between the source and destination. Further, we present analytical expressions for the outage probability and average achievable capacity. Simulation results validate the analytical expressions and show that the achievable capacity for the proposed PS protocol is better than the existing PS protocol.

Index Terms—adaptive power splitting, amplify and forward, capacity maximization, decode and forward, energy harvesting.

I. INTRODUCTION

COOPERATIVE communication (CC) systems can achieve higher performance than non-CC systems. A CC system comprises of a source, destination and a relay. In CC, the communication between source and destination is accomplished in two transmission phases. In the first transmission phase, the source transmits while in the second transmission phase the relay transmits to the destination. In conventional CC the relay uses its own battery energy to power its transmissions towards destination. This can result in exhaustion of the relay battery and due to this reason the relay may not participate in CC. This problem can be avoided if energy is harvested from wireless transmissions [1-13]. This is achieved at the relay through wireless energy harvesting (EH) from the source transmissions.

In general there are two types of protocols that are implemented at the relay to harvest energy from the source transmissions. These are time splitting (TS) protocol and power splitting (PS) protocol [1]. In PS protocol the received signal is divided into two parts during the whole receiving time. One part is fed to the EH circuitry while the other part is fed to the information processing circuitry. The implementation complexity of the PS protocol is less as compared to that of TS protocol.

The existing literature on PS protocol can be divided into two categories. One category deals with the fixed PS protocol [2-4] while the other category discusses adaptive PS protocol [5-9]. In fixed PS protocol, the PS ratio is constant over all the receiving times. However, in the adaptive PS protocol the PS ratio can vary over different receiving times. The adaptation of the PS ratio is dependent on the channel conditions between the source and the relay.

An adaptive PS protocol for amplify-and-forward (AF) relaying is presented in [5]. However, outage probability is not analyzed in their work. Further, the computation of their proposed PS ratio involves many arithmetic operations. For decode-and-forward (DF) relaying, in the conventional adaptive PS protocols the PS ratio also depends on the data rate of the communication [6], [8-9]. Therefore, they can work for fixed data rate communication systems only. In this paper we propose a novel adaptive PS protocol which maximizes the capacity of the CC system. The capacity maximization is achieved by maximizing the signal to noise ratio of the overall cooperative system. Further, we have provided novel analytical expressions for outage probability and average achievable capacity with the proposed PS ratio in AF and DF relaying systems. It is shown that the outage probability of the proposed scheme is same as the scheme in [6] while proposed scheme outperforms existing scheme in terms of average capacity.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a CC system which comprises of a source, destination and an EH relay. We assume that there is no direct connection available between source and destination and therefore communication is possible only with the help of the relay. The channel between source and relay is represented by $h_{s,r}$ while the channel between relay and destination is represented by $h_{r,d}$. The channel gains $|h_{s,r}|^2$ and $|h_{r,d}|^2$ follow exponential distribution with means $\lambda_{s,r}$ and $\lambda_{r,d}$, respectively. Further, the channels are assumed to be quasistatic which means that the channels remains constant over one transmission time while they can have different values over different transmission times. The relay can use either the AF or DF relaying protocol. The whole transmission time, $T$, is divided into two transmission phases. In the first transmission phase, the source transmits to the relay while in the second transmission phase the relay transmits to the destination. The relay uses PS protocol for EH and it uses only the harvested power during the first transmission phase to transmit during the second transmission phase. The signal that is sent to the relay during the first transmission phase is $\sqrt{(1-\rho)p_s|h_{s,r}|^2}n_r$, where $p_s$ is the transmit power of the source, $n_r$ is the additive white gaussian noise (AWGN) with variance $N_0$ and $0<\rho<1$ is PS ratio at the relay. The harvested power during the first transmission phase is given as follows [6]

$$y = \sqrt{(1-\rho)p_s|h_{s,r}|^2}n_r,$$

(1)

where $y$ is the harvested power. The harvested power is given as

$$P_h = \rho p_s|h_{s,r}|^2.$$

(2)

In this letter, we propose a novel power splitting (PS) protocol for energy harvesting (EH) cooperative communication systems. In our proposed system, the relay harvests energy from the source transmissions, by employing adaptive PS protocol, for powering the retransmissions to the destination. Unlike existing PS protocols, which are not optimal in terms of capacity maximization, our proposed adaptive PS protocol maximizes the achievable capacity between the source and destination. Further, we present analytical expressions for the outage probability and average achievable capacity. Simulation results validate the analytical expressions and show that the achievable capacity for the proposed PS protocol is better than the existing PS protocol.
The signal to noise ratio at the information processing circuitry of the relay is given as follows

\[ SNR_{s,r} = \frac{(1-\rho)P_s|h_{s,r}|^2}{N_0}. \]  

The capacity of the source to relay link can be obtained with the help of (3) as follows

\[ C_{s,r} = \frac{1}{2} \log \left( 1 + \frac{(1-\rho)P_s|h_{s,r}|^2}{N_0} \right). \]

Since the relay transmits with \( P_h \) power therefore the SNR of the relay to destination link corresponding to this transmit power is given as follows

\[ SNR_{r,d} = \frac{P_h|h_{r,d}|^2}{N_0} = \rho P_s|h_{s,r}|^2|h_{r,d}|^2. \]

In existing adaptive PS protocols the PS ratio is dependent on the rate of communication and \( |h_{s,r}|^2 \). For example for a data rate of \( C_{s,r} = R \) the PS ratio can be obtained from (4) as follows [6]

\[ \rho_{\text{exist}} = \max \left( 0, 1 - \frac{(2^R - 1)}{\gamma |h_{s,r}|^2} \right), \]

where \( \gamma = P_h/N_0 \). The reason for using this PS ratio is that if the channel conditions are good (i.e. if \( |h_{s,r}|^2 \geq \frac{(2^R - 1)}{\gamma} \)) then only the least required power which can ensure successful decoding at the relay is forwarded to the information processing circuitry while the rest of the received power is forwarded to the EH circuit. However, there is a problem with this adaptive PS ratio. If channel conditions are not good (i.e. if \( |h_{s,r}|^2 < \frac{(2^R - 1)}{\gamma} \) and \( 1 - \frac{(2^R - 1)}{\gamma |h_{s,r}|^2} < 0 \)) then the whole received signal will be forwarded to information processing circuitry even when it cannot decode it.

On the contrary, if a fixed value of \( \rho \) is used then it is possible that unnecessarily higher ratio of the received signal is fed to either EH circuit or information processing circuitry [6]. To address this problem we propose a novel PS protocol that maximizes the overall signal to noise ratio of the CC system.

III. PROPOSED PS PROTOCOL

We divide this section into two subsections. The first subsection proposes the PS protocol for DF relaying while the second subsection proposes PS protocol for AF relaying.

A. Decode-and-Forward relaying

For decode-and-forward relaying the overall SNR from source to destination can be written as follows

\[ SNR_{d,f} = \min \left( SNR_{s,r}, SNR_{r,d} \right) \]

\[ = \min \left( (1-\rho)\gamma |h_{s,r}|^2, \rho \gamma |h_{s,r}|^2 |h_{r,d}|^2 \right). \]

Lemma 1:

The following value of \( \rho \) maximizes the \( SNR_{d,f} \) in (7).

\[ \rho_{DF}^* = \frac{1}{1 + |h_{r,d}|^2}. \]

Proof:

We know that \( \min \left( SNR_{s,r}, SNR_{r,d} \right) = \min \left( (1-\rho)\gamma |h_{s,r}|^2, \rho \gamma |h_{s,r}|^2 |h_{r,d}|^2 \right) \) is maximized when all of its argument becomes equal. Therefore, the value of \( \rho_{DF}^* \) can be obtained by equating \( SNR_{s,r} \) to \( SNR_{r,d} \).

B. Amplify-and-Forward relaying

For amplify-and-forward relaying the received SNR at the destination can be written as follows [Eq. 4, 10]

\[ SNR_{d,f} = \frac{SNR_{s,r} SNR_{r,d}}{SNR_{s,r} + SNR_{r,d}} = \frac{(1-\rho)\gamma |h_{s,r}|^2 |h_{r,d}|^2}{(1-\rho) + \rho |h_{r,d}|^2}. \]

We have the following lemma for \( SNR_{d,f} \):

Lemma 2:

\[ SNR_{d,f}^* \] is a concave function of \( \rho \) and the value of \( \rho \) which maximizes the \( SNR_{d,f}^* \) is given as follows

\[ \rho_{AF}^* = \frac{1}{1 + |h_{r,d}|}. \]

Proof:

It can be observed that \( \frac{d^2 SNR_{d,f}^*}{d\rho^2} \) is negative for \( 0 < \rho < 1 \). Therefore, we conclude that \( SNR_{d,f}^* \) is a concave function of \( \rho \) for \( 0 < \rho < 1 \). In order to find the value of \( \rho \) which maximizes \( SNR_{d,f}^* \) we differentiate the \( SNR_{d,f}^* \) with respect to \( \rho \) and equate it to zero. After some simplifications we have the following possible solutions for \( \rho^* \)

\[ \rho_{AF}^* = \frac{1}{1 + |h_{r,d}|} \quad \text{or} \quad \rho_{AF}^* = \frac{1}{1 - |h_{r,d}|}. \]

Since \( \rho_{AF}^* = \frac{1}{1 + |h_{r,d}|} \) results in value of \( \rho_{AF}^* > 1 \) or \( \rho_{AF}^* < 0 \), therefore we pick \( \rho_{AF}^* = \frac{1}{1 + |h_{r,d}|} \) as the solution.

IV. OUTAGE PROBABILITY ANALYSIS

A. Decode-and-Forward relaying

The source to destination SNR for decode-and-forward relaying can be obtained by putting (8) into (7) as follows

\[ SNR_{d,f} = \frac{\gamma |h_{s,r}|^2 |h_{r,d}|^2}{1 + |h_{r,d}|^2}. \]

The corresponding expression for outage probability is given as follows

\[ P_{out}^{DF} = \frac{1}{\gamma_{th}} \left[ \exp \left( \frac{\gamma_{th} |h_{s,r}|^2 |h_{r,d}|^2}{1 + |h_{r,d}|^2} \right) \right] \gamma_{th}. \]

where \( \gamma_{th} \) depends on the required data rate (\( \gamma_{th} = 2^{2R} - 1 \)). Using the cumulative distribution function (CDF) of \( |h_{s,r}|^2 \) and [Eq. 3.324, 14] we can write \( P_{out}^{DF} \) as follows

\[ P_{out}^{DF} = \int_{0}^{\infty} \left[ 1 - \exp \left( \frac{-\gamma_{th} |h_{s,r}|^2 |h_{r,d}|^2}{1 + |h_{r,d}|^2} \right) \right] \lambda_{r,d} e^{-\lambda_{r,d} |h_{r,d}|^2} |h_{r,d}|^2 \gamma_{th}, \]

where \( \gamma_{th} \) depends on the required data rate (\( \gamma_{th} = 2^{2R} - 1 \)). Using the cumulative distribution function (CDF) of \( |h_{s,r}|^2 \) and [Eq. 3.324, 14] we can write \( P_{out}^{DF} \) as follows

\[ P_{out}^{DF} = \int_{0}^{\infty} \left[ 1 - e^{\frac{-\lambda_{r,d} |h_{s,r}|^2 |h_{r,d}|^2}{1 + |h_{r,d}|^2}} \right] \lambda_{r,d} e^{-\lambda_{r,d} |h_{r,d}|^2} |h_{r,d}|^2 \gamma_{th}, \]

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where $K_n(\cdot)$ denotes the modified Bessel function of second kind and $n$th order. Since the diversity order is defined as $\lim_{x \to \infty} \log \frac{P_{out}}{P_{in}}$ we can show that diversity order for DF scheme is 1 since $\lim_{x \to 0} xK_1(x) \to 1$ and $\lim_{x \to 0} e^{-x} \to 1 - x$.

### B. Amplify-and-Forward relaying

After putting the PS ratio $\rho_{AF}$ in (9) we can write the SNR for amplify-and-forward relaying as follows

$$SNR_{AF} = \frac{|h_{s,r}|^2 |h_{r,d}|^2}{(1 + |h_{r,d}|^2)^2}$$

The outage probability for amplify-and-forward relaying is defined as

$$P_{out} = P(\frac{|h_{s,r}|^2 |h_{r,d}|^2}{(1 + |h_{r,d}|^2)^2} < \gamma_{th}) = \int_0^\infty P(\frac{|h_{s,r}|^2}{|h_{r,d}|^2} < \frac{\gamma_{th}(1 + |h_{r,d}|^2)}{\gamma_{th}|h_{r,d}|^2}) \lambda_{r,d} e^{-\lambda_{r,d}|h_{r,d}|^2} d|h_{r,d}|^2$$

Using the CDF of $|h_{s,r}|^2$ and the approximation $e^{-2\gamma_{th}\frac{\lambda_{s,r}}{\gamma_{r,d}}|h_{r,d}|^2} \approx 1 - 2\gamma_{th}\frac{\lambda_{s,r}}{\gamma_{r,d}} |h_{r,d}|^2$ we can write

$$P_{AF} \approx 1 - \frac{1}{2} e^{-\gamma_{th}\frac{\lambda_{s,r}}{\gamma_{r,d}} |h_{r,d}|^2} K_1 \left( 2 \sqrt{\frac{\gamma_{th}\lambda_{s,r}}{\gamma_{r,d}} |h_{r,d}|^2} \right)$$

The above integral can be easily solved with the help of [Eq. 3.471.9, 14] as follows

$$P_{AF} \approx P_{out} = \frac{1}{2} e^{-\gamma_{th}\frac{\lambda_{s,r}}{\gamma_{r,d}} |h_{r,d}|^2} K_1 \left( 2 \sqrt{\frac{\gamma_{th}\lambda_{s,r}}{\gamma_{r,d}} |h_{r,d}|^2} \right)$$

The diversity order for AF scheme can be shown to be 1 since $\lim_{x \to 0} xK_1(x) \to 0$.

### V. AVERAGE CAPACITY ANALYSIS

#### A. Decode-and-Forward relaying

The average capacity for decode-and-forward relaying is given as follows

$$C_{DF} = E[|h_{r,d}|^2] \left[ \frac{1}{2} \int_0^\infty \log \left( 1 + \frac{|h_{s,r}|^2 |h_{r,d}|^2}{1 + |h_{r,d}|^2} \right) \frac{\lambda_{s,r} e^{-\lambda_{s,r} |h_{r,d}|^2}}{|h_{r,d}|^2} d|h_{r,d}|^2 \right]$$

$$\approx E[|h_{r,d}|^2] \left[ \frac{1}{2} \log \left( 1 + \frac{|h_{s,r}|^2 |h_{r,d}|^2}{1 + |h_{r,d}|^2} \right) \frac{\lambda_{s,r} e^{-\lambda_{s,r} |h_{r,d}|^2}}{|h_{r,d}|^2} \right]$$

where $E_1(.)$ is the exponential integral and $E_v[.|]$ denotes the expectation over random variable $v$. It is not possible to analytically solve (19) however we can use the following approximation for the exponential integral [15]

$$E_1[x] \approx 4\sqrt{2} \pi a_N \sum_{n=1}^{N+I+1} \sqrt{n} e^{-4a_n b_n x}$$

where $a_N = \frac{2N+2}{1 + \frac{N}{2} + 1}$, $a_I = \frac{2I}{1 + \frac{I}{2} + 1}$, $b_n = \frac{\cot(\theta_{in} - \theta_{in}) - \cot(\theta_{in})}{(N+1) + \pi}$, $b_0 = 0$, $\theta_a = \frac{\pi}{2N+2}$, $\theta_b = \frac{\pi}{2I+2}$.

Using this approximation we can write (19) as follows

$$C_{DF} \approx \frac{1}{2} \log \left( 1 + \frac{|h_{s,r}|^2 |h_{r,d}|^2}{1 + |h_{r,d}|^2} \right) \frac{\lambda_{s,r} e^{-\lambda_{s,r} |h_{r,d}|^2}}{|h_{r,d}|^2}$$

which can be easily solved with the help of [Eq. 3.324, 11] as follows

$$C_{DF} \approx \frac{1}{2} \log \left( 1 + \frac{|h_{s,r}|^2 |h_{r,d}|^2}{1 + |h_{r,d}|^2} \right) \frac{\lambda_{s,r} e^{-\lambda_{s,r} |h_{r,d}|^2}}{|h_{r,d}|^2}$$

The integral cannot be solved analytically. However, we can use (20) to get

$$C_{DF} \approx \frac{1}{2} \log \left( 1 + \frac{|h_{s,r}|^2 |h_{r,d}|^2}{1 + |h_{r,d}|^2} \right) \frac{\lambda_{s,r} e^{-\lambda_{s,r} |h_{r,d}|^2}}{|h_{r,d}|^2}$$

Comparing (24) with (22) it can be easily observed that $C_{DF} > C_{AF}$. This is because $K_1(x)$ decreases with increasing $x$ and $1 > \sqrt{\frac{\lambda_{r,d}}{\lambda_{r,d}}} |h_{r,d}|^2$ for $\lambda_{r,d} \geq 1$ and $\zeta \geq 0$.

#### C. Capacity comparison of the proposed DF with Existing DF

In the existing DF scheme [6] the overall SNR is $SNR_{DF} = \gamma \min \{|h_{s,r}|^2 (1 - \rho_{exist})^2, |h_{r,d}|^2 (1 - \rho_{exist})^2\}$. When $|h_{s,r}|^2 > \frac{2h_{r,d}}{1 + \rho_{exist}}$ there are two possibilities. Either $|h_{s,r}|^2 (1 - \rho_{exist}) > \rho_{exist}^2 |h_{r,d}|^2$ or $|h_{r,d}|^2 (1 - \rho_{exist}) > \rho_{exist}^2 |h_{s,r}|^2 |h_{r,d}|^2$. It can be easily verified that both of these possibilities result in $SNR_{DF} < SNR_{DF}^{exist}$. Therefore, we can say that the achieved capacity of the proposed scheme is higher than the existing DF scheme.
VI. SIMULATION RESULTS AND DISCUSSION

We provide two types of simulation results. First, we compare the outage probability of the proposed PS protocol with the existing PS protocol. Secondly, we compare the average capacity of the proposed PS protocol with the existing PS protocol in [6]. For simulations we use $\lambda_{s,r} = \lambda_{r,d} = 1$, $\gamma = 5 - 40$ dB, $R = [1, 3, 5]$ bps. Further, for the theoretical results of average capacity we use $I = N = 50$ in (22) and (24).

From Fig. 1 we can see that the outage probability of the proposed PS protocol for DF protocol is smaller than that of the AF protocol. This is because the SNR for the DF protocol is higher than the AF relaying protocol. Further, it can be observed that the outage probability of the proposed PS protocol is same as that of the existing PS protocol. This is because, for a fixed data rate, when an outage occurs for existing PS protocol then no other PS ratio can guarantee successful decoding at the destination. Since, the proposed PS protocol maximizes the SNR therefore we observe that the outage probability of the proposed PS protocol is same as that of the existing PS protocol.

![Outage probability comparison of proposed and existing scheme](image1)

Fig. 1. Outage probability comparison of proposed and existing scheme [6]. Theoretical results are marked with circles.

From Fig. 2 we can see that the proposed PS protocol outperforms existing PS protocol in terms of average capacity. This is because the PS ratio in the existing protocol is not optimal in terms of the overall SNR between the source and destination. On the other hand, since the proposed PS protocol maximizes the overall SNR between source and the destination therefore it performs better than the existing protocol in terms of average capacity. The improvement in capacity can also be explained with the reasoning provided in section V-C.

![Capacity comparison between proposed and existing scheme](image2)

Fig. 2. Capacity comparison between proposed and existing scheme [6]. Theoretical results are marked by circles.

In terms of implementation, we can see that the proposed PS protocol is easier to implement. This is because the proposed PS ratio depends on only one parameter (channel condition $|h_{r,d}|^2$) while the existing PS ratio depends on data rate ($R$), channel conditions ($|h_{s,r}|^2$) and transmit SNR $\gamma$. Further, no comparison with zero is required for calculating the PS ratio in the proposed PS protocol however it is required, according to (6), while calculating the PS ratio in the existing PS protocol.

VII. CONCLUSIONS

This letter presented a novel PS protocol for EH CC systems. Unlike the existing PS protocols, the PS ratio in the proposed protocol is only dependent on the channel conditions between the relay and destination. The outage probability of the proposed scheme is similar to that of the existing PS protocol. However, the proposed PS protocol outperforms the existing PS protocol in terms of average capacity. Further, analytical expressions and simulation results show that DF outperforms AF relaying scheme.

REFERENCES


